

FERMION SCATTERING AT A PHASE WAVE

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We study fermion reflection at a phase wave which is formed during a bubble collision in a first order phase transition. We calculate the reflection and the transmission coefficients by solving the Dirac equation with the phase wave background. Using the results we analyze the damping and the velocity of the wave.

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There have been many studies on the cosmological roles of the first-order phase transition which proceeds by nucleation and collisions of vacuum bubbles[1], especially in some inflation models[2].

If a global or a local symmetry is broken at the transition, there could be a phase wave, which has not been studied fully. In this paper, we study a fermion scattering at the phase wave.

At the nucleation, due to a finite correlation length, the two true vacuum bubbles that are separated by more than the correlation length or the horizon distance may have different phases. Such a possibility is a key ingredient for the Kibble mechanism[3].

If there is a phase difference between the two colliding bubbles, a pair of phase waves occur and propagate into each bubble. The phase wave [4, 5] is a mechanism by which the bubbles get their new phase values determined by the so-called “geodesic rule” in a first [5] or a second[6] order phase transition.

The energy of the colliding bubble wall turns into that of the modulus wall and the phase wave. If the phase difference of the two bubbles is of order 1, the phase wave carries away most of the energy of the bubble wall [4]. So the phase wave can be a very energetic phenomenon as the bubble wall is.

Furthermore, investigating the particle scattering at a moving bubble wall is important to study the bubble kinetics and cosmology. For example, to calculate the velocity and the width of the electro-weak bubbles[7] and the CP violat-

ing charge transport rate by the wall for baryogenesis [8], one should know the reflection coefficient of fermions(*e.g.* top quarks) scattering at the wall.

However, there have been a few works about the dynamics and cosmological roles of the phase wave compared with the bubble walls.

So, in this paper, we investigate the interaction between the phase wave and fermions. In the cosmological phase transition, a propagating phase wave may collide with the fermions in the plasma.

Consider a complex scalar field $\phi(x) = \rho(x)\exp(i\theta(x))$ whose Lagrangian density is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi - V(\rho), \quad (1)$$

where $V(\rho)$ is a potential appropriate for a first-order phase transition.

Since the potential depends only on the amplitude ρ , one may choose an arbitrary phase for the minimum of the potential, $\langle\phi\rangle$. Of course, this is due to the global $U(1)$ symmetry the theory possesses. In general, we can make the true vacuum to lie along the direction of the real part of ϕ , because the phase of ϕ can be absorbed by redefining the phase of the fermion field. However, if $\langle\phi\rangle$ is space-time dependent, we can not globally rotate out the phase which may play physical roles.

From \mathcal{L} we obtain the equation of motion for θ ;

$$\partial^\mu\partial_\mu\theta + \frac{2}{\rho}\partial^\mu\theta\partial_\mu\rho = 0. \quad (2)$$

Without loss of generality, we can choose the two phases for the colliding

bubbles as zero and $2\Delta\theta(<\pi)$, respectively. Then, by the geodesic rule, the overlapping region of the two bubbles should have $\Delta\theta(<\pi/2)$.

We consider the case where the width of the phase wave (d) is much smaller than the radius of curvature (r) of the phase wave. In this case we can treat our problem as one dimensional. We choose z axis as the direction normal to the plane of the phase wave, and assume that momentum of the incident fermion is parallel to the z axis. The solution for the more general momentum can be found by appropriately Lorentz boosting the one dimensional solution.

A possible solution of eq.(2) is

$$\theta(z, t) = \Delta\theta[1 + \frac{1}{2}f(z + vt) + \frac{1}{2}f(z - vt)], \quad (3)$$

where $f(z)$ is some profile function which has an asymptotic value ± 1 at $z = \pm\infty$, respectively.

The second term indicates the left moving phase wave($-z$ direction) with velocity v relative to the plasma, while the third term is the right moving one ($+z$ direction). Since $\partial_\mu\rho = 0$ for the phase wave, v is the speed of light c . However, eq.(2) ignored the pressure by the fermion scattering which is just what we are going to study in this paper. So we assume $|v| < c$.

To calculate the reflection coefficient of the fermion we choose the rest frame of the left moving phase wave ($v = 0$) and assume that the incident fermion comes from the left. (See Fig.1.) When the typical wave length and the mean free path of the fermion are much larger than the phase wave width $\sim 1/m_0$, we

can approximate $f(z)$ as $2\Theta(z) - 1$ with step function $\Theta(z)$, i.e., $\theta(z) = \Delta\theta$ when $z \geq 0$ and zero for $z < 0$.

Consider a Lagrangian density for the fermion Ψ

$$L_\Psi = \bar{\Psi}_L i\partial \Psi_L + \bar{\Psi}_R i\partial \Psi_R - (h\bar{\Psi}_L \Psi_R \phi + h^* \bar{\Psi}_R \Psi_L \phi^*) \quad (4)$$

with the (real) Yukawa coupling constant h . In the rest frame of the phase wave, Ψ acquires position dependent mass $m(z) = h < \phi(z) >$. Therefore we get the equation of motion for Ψ from L_Ψ ;

$$(i\partial - m(x)P_R - m^*(x)P_L)\Psi = 0, \quad (5)$$

where $P_{R,L}$ are the chirality projection operators.

We adopt the following ansatz[9, 10]

$$\begin{aligned} \Psi(t, z) &= (i\partial + m^*(z)P_R + m(z)P_L)e^{-i\sigma Et}\psi(k, z) \\ &= (\sigma E\gamma^0 + i\gamma^3\partial_z + m_R(z) - im_I(z)\gamma_5)e^{-i\sigma Et}\psi(k, z), \end{aligned} \quad (6)$$

where a field ψ satisfying following the Klein-Gordon-like equation is introduced;

$$(E^2 + \partial_z^2 - |m(z)|^2 + im'_R(z)\gamma^3 - m'_I(z)\gamma_5\gamma^3)\psi(z) = 0. \quad (7)$$

Here prime denotes the derivative with z , $m(z) = m_R(z) + im_I(z)$ and σ is $+1$ and -1 for fermion and antifermion, respectively.

If $m'(z) \neq 0$, there is an additional potential term proportional to the spatial change of $m(z)$, which is the origin of the scattering. For our case, there is a delta function type potential at $z = 0$.

Let us find momentum eigen states. We expand ψ in the eigenspinors of γ^3 with eigenvalue $\pm i$ satisfying $\gamma^3 u_{\pm}^s = \pm i u_{\pm}^s$, where $u_{\pm}^1 = 1/\sqrt{2}(1, 0, \pm i, 0)^T$ and $u_{\pm}^2 = 1/\sqrt{2}(0, 1, 0, \mp i)^T$ with spin index $s = 1, 2$. (We use here the same conventions for the γ matrices and u_{\pm}^s in ref.[9].) Note that $\gamma^0 u_{\pm}^s = u_{\mp}^s$.

Since $m(z) = m_0 e^{i\theta(z)}$, $m_R = m_0$ and $m_I = 0$ for $z < 0$ (region I) and $m_R(z) = m_0 \cos(\Delta\theta) \equiv m_c$ and $m_I(z) = m_0 \sin(\Delta\theta) \equiv m_s$ for $z \geq 0$ (region II).

From now on we will consider the case of particle ($\sigma = 1$). For $z \neq 0$, $m'(z) = 0$ and eq.(7) becomes a free Klein-Gordon equation. Then, the most general right moving solution to eq.(7) can be represented as a combination of u_{+}^s and u_{-}^s , i.e.,

$$\psi = e^{ikz} \sum_s \sum_{\pm} C_{\pm}^s u_{\pm}^s, \quad (8)$$

where $k = \sqrt{E^2 - m_0^2}$ and C_{\pm}^s are some constants.

Inserting ψ into eq.(6) we get Ψ for $z < 0$;

$$\Psi = e^{-iEt+ikz} \sum_s \{C_{+}^s (Eu_{-}^s + (m_0 - ik)u_{+}^s) + C_{-}^s (Eu_{+}^s + (m_0 + ik)u_{-}^s)\}. \quad (9)$$

These terms are not independent in each other in u_{\pm}^s basis, so we can set $C_{-}^s = 0$ without loss of generality. Therefore,

$$\psi(z) = e^{ikz} \sum_s C_{+}^s u_{+}^s. \quad (10)$$

Similarly, for the left moving wave we can choose Ψ proportional to $(Eu_{+}^s + (m_0 - ik)u_{-}^s)$. So for the region I , the general solution can be given by

$$\begin{aligned} \Psi_I = e^{-iEt} \sum_s C_{+}^s \{e^{ikz} [(m_0 - ik)u_{+}^s + Eu_{-}^s] \\ + R_s e^{-ikz} [Eu_{+}^s + (m_0 - ik)u_{-}^s]\}. \end{aligned} \quad (11)$$

The first term is a incoming wave and the second term is a reflected one. In the region II only a transmitted wave exists;

$$\Psi_{II} = e^{-iEt+ikz} \sum_s T_s C_+^s \{ (E - m_s(-)^s) u_-^s + (m_c - ik) u_+^s \}. \quad (12)$$

Here, R_s and T_s are some spin-dependent coefficients.

Now, equating Ψ_I and Ψ_{II} at $z = 0$ and comparing each coefficients of u_{\pm}^s , we get two algebraic equations for R_s and T_s ;

$$\begin{aligned} R_s &= \frac{-E + (E - (-)^s m_s) T_s}{m_0 - ik}, \\ T_s &= \frac{m_0 - ik + E R_s}{m_c - ik}, \end{aligned} \quad (13)$$

which has a solution

$$\begin{aligned} R_s &= \frac{1}{D} [E(m_0 - m_c) - (-)^s m_s (m_0 - ik)], \\ T_s &= \frac{1}{D} [(m_0 - ik)^2 - E^2], \end{aligned} \quad (14)$$

where $D = (m_c - ik)(m_0 - ik) - E(E - (-)^s m_s)$.

From Ψ_I and Ψ_{II} we get a vector current $j^3 = \bar{\Psi} \gamma^3 \Psi$ for the region I and II .

$$\begin{aligned} j_I^3 &= 2kE \sum_s |C_+^s|^2 (1 - |R_s|^2), \\ j_{II}^3 &= 2kE \sum_s |C_+^s|^2 |T_s|^2 (1 - \frac{(-)^s m_s}{E}). \end{aligned} \quad (15)$$

During this calculation the orthogonality condition $\bar{u}_{\pm}^s u_{\mp}^{s'} = \delta_{ss'}$ and $\bar{u}_{\pm}^s u_{\pm}^{s'} = 0$

are useful. Here, from eq.(14)

$$\begin{aligned} |R_s|^2 &= \frac{2m_0(m_0 - m_c)(1 - (-)^s \frac{m_s}{E})}{F}, \\ |T_s|^2 &= \frac{4k^2}{F} \end{aligned} \quad (16)$$

with $F = 2(2k^2 + m_0(m_0 - m_c))(1 - (-)^s m_s/E)$. We have used the relations $E^2 = k^2 + m_0^2$ and $m_s^2 + m_c^2 = m_0^2$ repeatedly.

So the reflection and the transmission coefficients are given by

$$\begin{aligned} R &= \frac{\sum_s |C_+^s|^2 |R_s|^2}{\sum_s |C_+^s|^2} = \frac{m_0(m_0 - m_c)}{2k^2 + m_0(m_0 - m_c)}, \\ T &= \frac{\sum_s |C_+^s|^2 |T_s|^2 (1 - (-)^s \frac{m_s}{E})}{\sum_s |C_+^s|^2} = \frac{2k^2}{2k^2 + m_0(m_0 - m_c)}. \end{aligned} \tag{17}$$

It is clear that the unitarity follows from eq.(17), i.e., $R + T = 1$. We can get the coefficient for an antifermion by substituting $-E$, $-k$ and $C_+^{'s}$ (corresponding to C_+^s) instead of E , k and C_+^s respectively. Since R and T are not changed by these substitutions, we can say that the fermion and the antifermion reflect at the phase wave in a CP conserving way. It is contrary to the bubble wall case where a position dependent phase of ϕ generally induces a CP violating reflection.

Since R and T depend on $\cos(\Delta\theta)$, there is no difference between $\Delta\theta$ and $-\Delta\theta$. It also means that R and T are not dependent on the incoming direction of the fermion (left or right).

Now let us find approximate R for $\Delta\theta \ll 1$. In this case, $m_c \simeq m_0(1 - \Delta\theta^2/2)$ and $m_s \simeq m_0\Delta\theta$. Gathering all terms up to $O(\Delta\theta^2)$ we find

$$R \simeq \frac{m_0^2 \Delta\theta^2}{4k^2}. \tag{18}$$

This implies that the particles with momentum $k \ll k_c \equiv m_0\Delta\theta/2$ will totally

reflect at the phase wave and transfer the momentum $2k$ to the wave.

We also confirm this result (18) by solving eq.(5) and eq.(7) in a second-order perturbation [11] when the width of the wave d is finite. There we checked that R in eq.(18) is the leading order term at $d \rightarrow 0$ limit, expanding $e^{i\theta} \simeq 1 + i\theta - \theta^2/2$ and finding solution up to the second order,

From eq.(17) we find that the definition for k_c is also reasonable even for $\Delta\theta \simeq 1$. Hence we will approximate $R(k)$ as $\theta(k_c - k)$.

When there is no viscosity, as the phase wave expands, it should lose its gradient energy $(\partial_\mu \theta)^2$ and its width d becomes thicker and thicker, because, contrary to the bubble wall case, there is no net pressure from the vacuum energy difference across the phase wave[4].

So it is possible that the particle scattering may reduce the velocity of the wave and even stop it.

The viscosity by this reflection could be calculated by the thermally averaged momentum transfer $\langle P \rangle$.

From the energy momentum relation $E^2 = k_\perp^2 + k_z^2 + m_0^2$, where $k_\perp^2 = k_x^2 + k_y^2$, we find $k_z dk_z = E dE$. In thermal equilibrium states, there are $n(E) d^3k / (2\pi)^3 |k_z/E|$ particles per unit wall area in unit time providing a momentum transfer $2k_z$ [12]. Here $n(E) = (e^{E/T} + 1)^{-1}$ is the Fermi distribution function at the temperature T .

Assuming $v > 0$, we obtain

$$\begin{aligned}\langle P \rangle = & \int \frac{d^2 k_\perp}{(2\pi)^3} \left\{ \int_0^\infty dk_z 2k_z R(k_z) \frac{k_z}{E} n[\gamma(E + vk_z)] \right. \\ & \left. + \int_{-\infty}^0 dk_z 2k_z R(k_z) \frac{-k_z}{E} n[\gamma(E + vk_z)] \right\}. \quad (19)\end{aligned}$$

where $d^2 k_\perp = dk_x dk_y$. The first term corresponds to the pressure by the fermions with $k_z > 0$, while the second term corresponds to that with $k_z < 0$. Changing integration variable from k_z to $-k_z$ in the second term we obtain

$$\langle P \rangle = \int \frac{d^2 k_\perp}{(2\pi)^3} \int_0^\infty dk_z 2\theta(k_c - k_z) \frac{k_z^2}{E} \{ n[\gamma(E + vk_z)] - n[\gamma(E - vk_z)] \} \quad (20)$$

Fig. 2. displays a numerical result of this integration. $\langle P \rangle$ increases linearly as v increases, and then goes to zero as v approaches to 1.

Now we assume that the velocity of the wave is nonrelativistic ($v \ll 1$). Then $n(\gamma(E + vk_z)) \simeq n(E) + vk_z \frac{\partial n(E)}{\partial E} = n(E) + v\beta k_z n(E)(n(E) - 1)$. Changing integration variables from $d^3 k$ to $2\pi E dE dk_z$ and assuming $\beta E \gg 1$ give

$$\begin{aligned}\langle P \rangle &= -v \frac{\beta}{\pi^2} \int_{m_0}^\infty dE e^{-\beta E} \int_0^{\sqrt{E^2 - m_0^2}} dk_z \theta(k_c - k_z) k_z^3 \\ &= -v \frac{\beta}{\pi^2} \left\{ \int_{m_0}^{E_c} dE e^{-\beta E} \int_0^{\sqrt{E^2 - m_0^2}} dk_z \theta(k_c - k_z) k_z^3 \right. \\ &\quad \left. + \int_{E_c}^\infty dE e^{-\beta E} \left[\int_0^{k_c} dk_z \theta(k_c - k_z) k_z^3 + \int_{k_c}^{\sqrt{E^2 - m_0^2}} dk_z \theta(k_c - k_z) k_z^3 \right] \right\}, \quad (21)\end{aligned}$$

where $E_c^2 \equiv m_0^2 + k_c^2 = m_0^2(1 + \Delta\theta^2/4)$ and we used the approximation $n(E) \simeq \exp(-\beta E)$. We have separated the integration regions to drop the third term.

Then,

$$\begin{aligned}\langle P \rangle &\simeq -v \frac{\beta}{4\pi^2} \int_{m_0}^{E_c} dE e^{-\beta E_c} (E^2 - m_0^2)^2 - v \frac{k_c^4}{4\pi^2} e^{-\beta E_c} \\ &\simeq -v \frac{k_c^4}{4\pi^2} e^{-\beta E_c},\end{aligned}\tag{22}$$

because the first term is of order $(\Delta\theta)^6$, while the second term is of order $(\Delta\theta)^4$.

For $\theta = \theta(r - vt)$ the Newtonian equation for r is given by[13]

$$\sigma \frac{d^2 r}{dt^2} = -\frac{2\sigma}{r} + \langle P \rangle\tag{23}$$

where v is now dr/dt . This equation can be derived by considering the force acting on the unit spherical area of the wave with the ‘radius’ r and ‘mass’ density σ .

The surface energy density σ is related to the energy momentum tensor by

$$\sigma = \int dz T_0^0 = \int dz \frac{1}{2} (\partial_z \phi)^2 \simeq \frac{\rho^2}{d}.\tag{24}$$

Hence for nonrelativistic and planar approximation($r \rightarrow \infty$)

$$\sigma \frac{d^2 r}{dt^2} = \sigma \frac{dr}{dt} v \simeq -v \frac{k_c^4}{4\pi^2} e^{-\beta E_c},\tag{25}$$

which has a solution

$$v = v_0 \exp\left[-\frac{k_c^4}{4\pi^2 \sigma} \exp(-\beta E_c) t\right],\tag{26}$$

where v_0 is an initial velocity.

Hence the phase wave stops in a time scale $t_{stop} \equiv \frac{4\pi^2 \sigma}{k_c^4} e^{\beta E_c}$ and the wave travels the distance

$$\Delta r \simeq v_0 t_{stop} [1 - \exp(-t/t_{stop})]\tag{27}$$

during t .

The bubble nucleation rate per unit time and volume is $\Gamma \simeq T^4 e^{-A}$ and $A \simeq 4 \ln M_P / T$ [14]. The average bubble distance and the bubble percolation time scale is $t_{bub} \sim \beta^{-1}$, where $\beta = \gamma A H$. Here the potential form dependent factor $\gamma = d \ln A / d \ln T$ is generally $O(1) \sim O(10^2)$ [15].

For example, if $T \simeq 10^2 \text{ GeV}$, $A \simeq 10^2$ and $t_{bub} \simeq 10^2 H^{-1} \simeq 10^2 M_P / T^2$. On the other hand, if all the coupling constants are not too smaller than unity, then $m_0 \sim T$, and $t_{stop} \sim T^{-1} e^{\beta E_c}$. Hence, for β not too smaller than E_c , t_{stop} should be shorter than t_{bub} . Generally if $T \ll e^{-\beta E_c} M_P$, the phase wave may stop before it reaches the other side of the bubble wall.

If these stopped phase waves persist long time, their energy density may give rise to a problem similar to the domain wall problem [3]. Even if some ignored mechanisms (for example surface tension) make the wave disappear, they take a role only within the horizon at that time and hence the problem could not be removed completely. So in this view point, it might be preferable to have a non-degenerate unique vacuum for $\langle \phi \rangle$.

When the Yukawa coupling h is complex, defining $m_c \equiv m_0 (Re(h) \cos(\Delta\theta) - Im(h) \sin(\Delta\theta))$ and $m_s \equiv m_0 (Re(h) \sin(\Delta\theta) + Im(h) \cos(\Delta\theta))$ gives the same result in eq.(17).

If there is another scalar field χ interacting with ϕ through $(\lambda/2) \lambda |\phi|^2 |\chi|^2$, χ gets a constant mass $\lambda^{1/2} \rho$ and satisfies Klein-Gordon equation. Hence the scalar

field χ does not reflect at the phase wave.

The origin of fermion scattering at the phase wave is a position dependent phase change of its mass, while that of the bubble wall is a change of absolute mass and, therefore, a momentum change across the wall.

Our results may be also applied to detection of a spatial phase change and its velocity v , if any, in the universe or a condensed matter. For example, by using eq.(18) and observing the difference between the two R 's for electron beams with the opposite incoming directions. (say, from region I and II in fig.1., respectively.) Each electron beams should have different k in the phase wave rest frame moving with v , and have different R .

The results, so far, are for a global symmetry without gauge fields. Including gauge fields may give more complicated and interesting results.

In summary, we calculated the reflection coefficient of the fermion scattering off the phase wave, which could be a first step toward a comprehensive understanding of the phase wave dynamics in the hot plasma. As an example, we investigated the damping and the velocity of the wave due to this scattering, and found that this damping could even stop the wave in some cases.

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FIG.1.

$\theta(z)$ in the phase wave rest frame. Ψ_I consists of the incoming and the reflected waves and Ψ_{II} is the transmitted wave.

FIG.2.

The damping pressure $\langle P \rangle$ in units of m_0^4 versus the velocity of the phase wave v for $\Delta\theta = 0.1$ and $T = 2m_0$. The solid curve shows the results of eq.(20) and the dotted line shows the results of eq.(22).

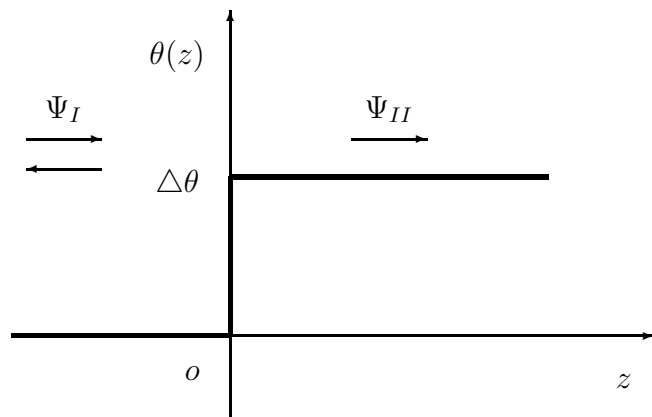


Fig. 1.

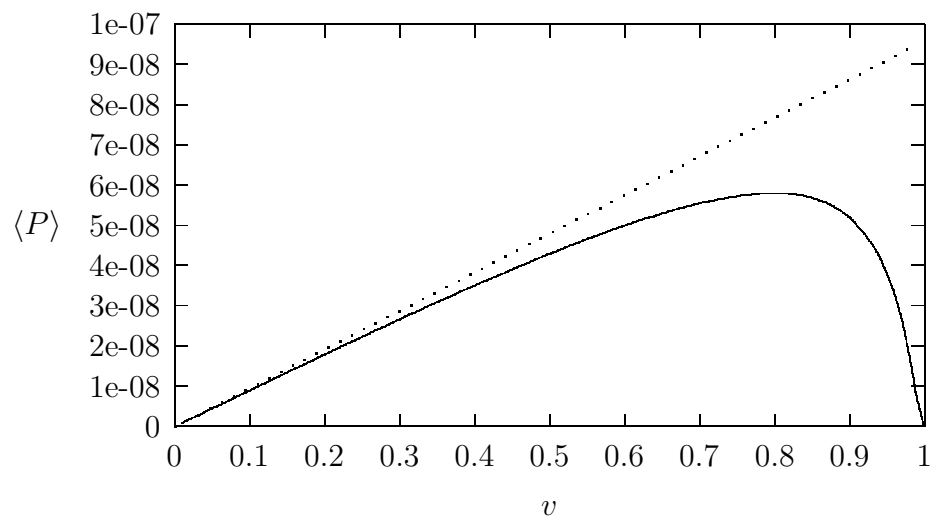


Fig. 2.